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Lepto-quark unification scale in SUSY SO(10) GUT

N Nimai Singh and S Biramani Singh
Department of Physics, Gauhati University, Guwahati-781 014, Assam, India

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Abstract : We comment on the sensitivity of the running quark masses to the scale of third generation Yukawa couplings unification in the minimal supersymmetric standard model (MSSM). With slight variations of the input running quark masses, Yukawa couplings unification can be achieved at both intermediate scale ($M_I = 1.4 \times 10^{12}$ GeV) and GUT scale ($M_U = 1.87 \times 10^{16}$ GeV). This scenario conforms with the pattern of the symmetry breaking of the SUSY SO (10) GUT with or without intermediate symmetry.

Keywords : Yukawa couplings, SUSY SO(10) GUT, intermediate scale

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1. Introduction

During the last one decade the question of third generation Yukawa couplings unification in the minimal supersymmetric standard model (MSSM) embedded in GUTs like SUSY SO(10) model, has been studied mainly from two angles : (i) the low-energy prediction of either top or bottom quark mass, and m_b/m_τ ratios from the boundary condition of Yukawa couplings unification at GUT scale as starting point [1–4], and (ii) the prediction of the third generation Yukawa couplings unification scale through Renormalization Group Equations (RGEs) of the Yukawa couplings evolving from the electroweak scale (M_Z), using the experimental input values of $m_{\rm top}$, m_b and m_τ [5–8]. The second option does not fully address the question whether Yukawa couplings unification can occur at an intermediate scale (M_I) or at the GUT scale (M_U) [2,8]. This aspect is also related to the symmetry breaking pattern of SUSY SO(10) model with or without an intermediate symmetry breaking scale. The possible existence of an intermediate symmetry scale around $M_I \sim 10^{11} - 10^{12}$ GeV corresponding to (B–L) symmetry breaking, has been studied thoroughly in SUSY SO(10) model [9–12], and such a scale is particularly important for achieving small neutrino masses through seesaw mechanism necessary to understand solar and atmospheric neutrino flux and/or the dark matter of the universe [13]. The Yukawa coupling (h_N) of the right-handed Majorana neutrino (if they exist) with mass M_N substantially lighter than the lepton number (L) or (B–L) violating scale (M_I), affects the RGEs due to the presence of an extra term in the superpotential of the MSSM : $h_N \overline{\psi}_L \nu_R \Phi_u$, $\mu \geq M_N$, where ψ_L is the lepton doublet [13–15]. The right-handed neutrinos decouple at $\mu < M_N$, and $M_N << M_X$, one usually considers the renormalization effect due to the Yukawa interactions of neutrino from the region of momentum $M_N \leq \mu \leq M_X$. In such case the GUT boundary conditions [13] are taken as $h_{\rm top} = h_N$, $h_b = h_\tau$.

In this paper, we address the second option regarding the issue related to the Yukawa couplings unification at both the intermediate and GUT scales in MSSM, and the sensitivity of the input values of top and bottom quark masses. In Section 2, we outline the formalism and collect all the relevant RGEs, while in Section 3, the numerical results and conclusion are given.

2. Evolution equations of the Yukawa couplings

The renormalization group equations (RGEs) of the Yukawa coupling evolution relevant for the present investigation, in MSSM are given by

$$\frac{dh_f(t)}{dt} = 0h_f(t) + Th_f(t), \quad f = \text{top}, b, \tau, N, \tag{1}$$

where the two-loop contributions ($Th_f(t)$) are given in Ref. [2] and the one-loop contributions ($0h_f(t)$) are collected here [2,14,15],

$$16\pi^2 0h_{\rm top} = h_{\rm top}\left[6h_{\rm top}^2 + h_b^2 + h_N^2\theta_N - \sum_{i=1}^{3} c_i g_i^2\right], \tag{2}$$

$$16\pi^2 0h_b = h_b\left[6h_b^2 + h_\tau^2 + h_{top}^2 - \sum_{i=1}^{3} c_i' g_i^2\right], \tag{3}$$

$$16\pi^2 0h_\tau = h_\tau\left[4h_\tau^2 + 3h_b^2 + h_N^2\theta_N - \sum_{i=1}^{3} c_i'' g_i^2\right], \tag{4}$$

$$16\pi^2 0h_N = h_N\theta_N\left[4h_N^2 + 3h_{\rm top}^2 + h_\tau^2 - \sum_{i=3}^{3} c_i^N g_i^2\right], \tag{5}$$

where the coefficients are

$c_i = (13/15, 3, 16/3)$, $\quad c_i' = (7/15, 3, 16/3)$,

$c_i'' = (9/5, 3, 0)$, $\quad c^N = (3/5, 3, 0)$.

The two-loop RGEs for the gauge couplings are also given below [2] :

$$(16\pi^2)^2 \frac{dg_i(t)}{dt} = g_i^3 \; 16\pi^2 b_i + \sum_{i=1} b_{ij} g_i^2 - \sum_{j=\mathrm{top},b,\tau} a_{ij} h_j^2 \tag{6}$$

$$b_i = (33/5, 1, -30), \; b_{ij} = \begin{matrix} & 5.4 & 17 \\ 1.8 & 25 & 24 \\ 2.2 & 9 & 14 \end{matrix}\Bigg], \; a_{ij} = \begin{bmatrix} 26/5 & 14/5 & 18/5 \\ 6 & 6 & 2 \\ 4 & 4 & 0 \end{bmatrix} \tag{7}$$

where the scales of evolution are defined as $t = \ln(\mu/1\mathrm{GeV})$, $t_0 = \ln(m_{\mathrm{top}}/1\mathrm{GeV})$, $t_N = \ln(M_N/1\mathrm{GeV})$, $\theta_N = \theta(t - t_N) = 1$ (or 0) when $\mu >$ (or $<$) M_N; and g_1, g_2 and g_3 correspond to the gauge couplings of $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ of the MSSM assumed to hold for $\mu \geq m_{\mathrm{top}} = M_{\mathrm{SUSY}}$. The scale dependence of the Yukawa couplings at the scale of top quark running mass, can be expressed through the fermion running masses and free parameter $\tan\beta = v_u/v_d$, $v = \sqrt{(v_u^2 + v_d^2)} = 174$ GeV,

$$m_{\mathrm{top}}(t_0) = h_{\mathrm{top}}(t_0) \frac{v \tan\beta}{\sqrt{(1 + \tan^2\beta)}}, \tag{8}$$

$$m_b(t_0) = h_0(t_0) \frac{}{\sqrt{(1 + \tan^2\beta)}}, \tag{9}$$

$$m_\tau(t_0) = h_\tau(t_0) \frac{}{\sqrt{(1 + \tan^2\beta)}}, \tag{10}$$

$$m_b(t_0) = m_b(t_b)/\eta_b, \quad m_\tau(t_0) = m_\tau(t_\tau)/\eta_\tau. \tag{11}$$

where $t_b = \ln(m_b/1\ \mathrm{GeV})$, $t_\tau = \ln(m_\tau/1\ \mathrm{GeV})$. In eqs. (8–10), $m_{\mathrm{top}}(m_{\mathrm{top}})$, $m_b\ (t_b)$, and $m_\tau\ (t_\tau)$ refer to the running masses of the top, bottom and τ lepton; and η_i are the QCD-QED rescaling factors defined [2,3] as $m_i(t_0) = m_i(m_i)/\eta_i$, $i = b, \tau$. The values of $g_1(t_0)$, $g_2(t_0)$, $g_3(t_0)$ at the top quark mass scale (t_0) are estimated using two-loop RGEs, from the CERN-LEP measurements at $M_Z = 91.18$ GeV [16,17],

$$\begin{aligned} \sin^2\theta_\omega(M_Z) &= 0.2310 \pm 0.0003, \\ \alpha_s(M_Z) &= 0.118 \pm 0.004, \\ \alpha^{-1}(M_Z) &= 127.9 \pm 0.1. \end{aligned} \tag{12}$$

We make use of the running masses of the top, bottom and τ in eqs. (9–10) through eq. (11). For light quarks and leptons, the differences between the pole (physical/experimental) and the running masses at low energies are negligible whereas for heavy quarks (top, b), the differences are significant. For τ-lepton, the running mass is taken as the experimental mass $m_\tau = m_\tau(t_\tau) = 1.785$ GeV, but for heavy flavours like top and bottom quarks, we employ the two-loop formula [18,19]

$$m_{\mathrm{top}}^{\mathrm{Pole}} = m_{\mathrm{top}}(m_{\mathrm{top}}) \; 1 + \frac{4}{3}\left(\frac{\alpha_s(m_{\mathrm{top}})}{-}\right) + 8.28\left(\frac{\alpha_s(m_{\mathrm{top}})}{\pi}\right. \tag{13}$$

which gives $m_{top}(m_{top}) = 166.5 \pm 5.5$ GeV corresponding to $m_{top}^{Pole} = 175.6 \pm 5.5$ GeV [18].

$$m_b^{Pole} = m_b(m_b)\left[1+\frac{4}{3}\frac{\alpha_s(m_b)}{\pi}+\left(16.11-1.04\sum_k\left(1-\frac{m_{f_k}}{m_b}\right)\right)\times\left(\frac{\alpha_s(m_b)}{\pi}\right)^2\right]. \quad (14)$$

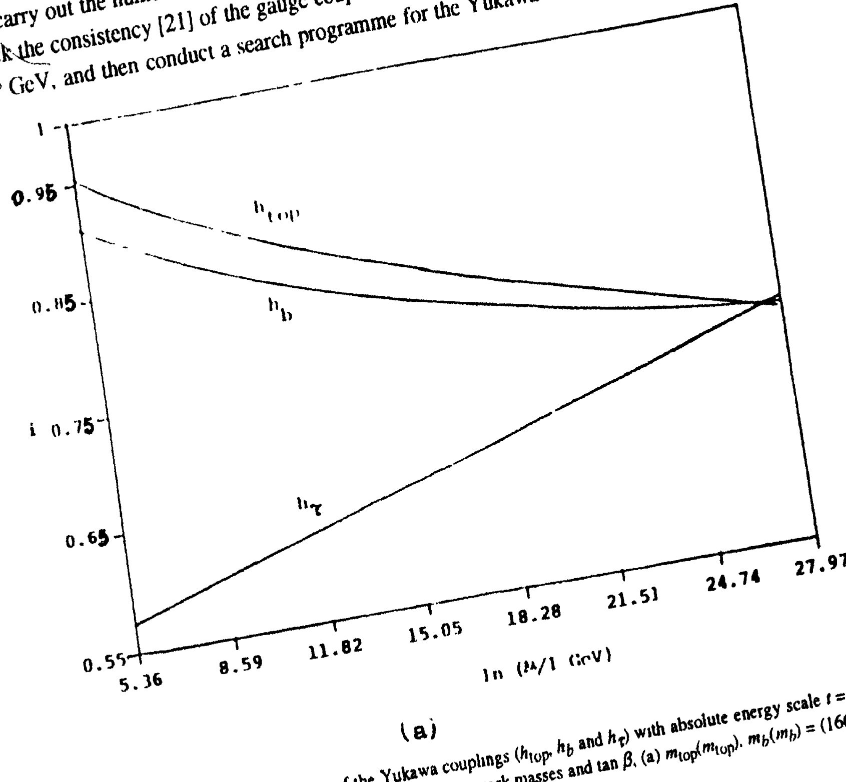
where the sum over k extends over all flavours f_k lighter than b quark [19]. For reasonable value of $\alpha_s(m_b) = 0.22$, eq. (14) reduces to $m_b^{Pole} = m_b(m_b)$ [1 + 0.09 + 0.06] where the second and third terms refer to one-loop and two-loop contributions respectively [19]. For a reasonable range of $m_b^{Pole} = (5.0-4.7)$ GeV, we get the value of the running mass $m_b(m_b) = (4.3–4.2)$ GeV. In eqs. (13) and (14) we have neglected small contributions from QED and Higgs sector [20]. The values of the QCD–QED rescaling factors estimated from three-loop QCD and one-loop QED, are given by

$$\eta_b = 1.540^{+0.095}_{-0.087}, \quad \eta_\tau = 1.017 \pm 0.001, \quad (15)$$

where the uncertainties are estimated from input values, eq. (12).

3. Results and conclusion

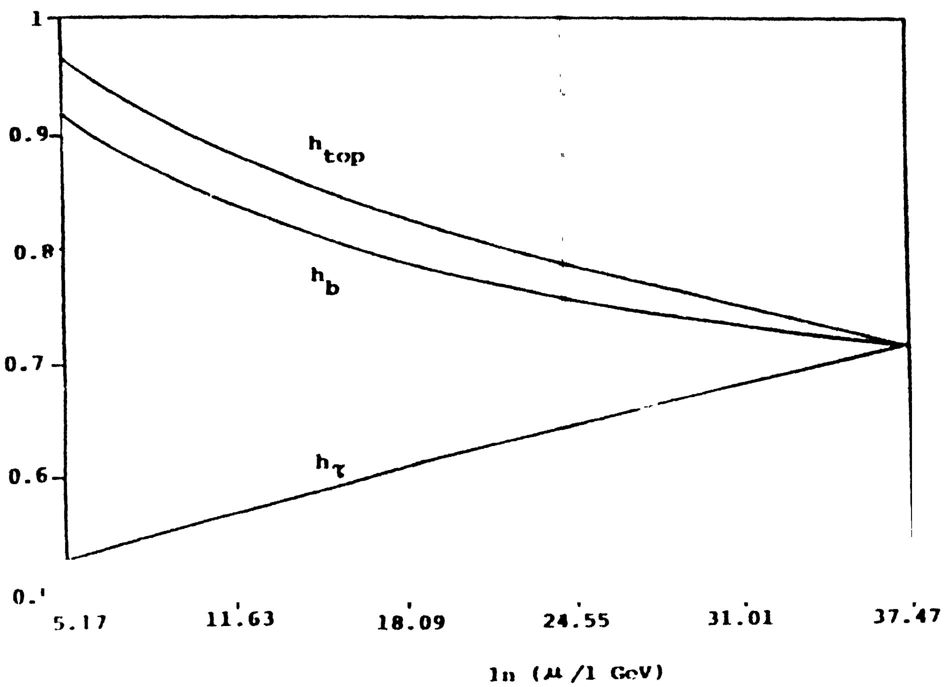
We carry out the numerical analysis of RGEs, eqs. (1) and (6), at two-loop level. First we check the consistency [21] of the gauge couplings unification at the GUT scale $M_U = 1.8 \times 10^{16}$ GeV, and then conduct a search programme for the Yukawa couplings unification for



(a)

Figure 1. Variation of the Yukawa couplings (h_{top}, h_b and h_τ) with absolute energy scale t = ln (μ/1 GeV) for input values of running quark masses and tan β, (a) $m_{top}(m_{top})$, $m_b(m_b)$ = (166.5, 4.2) GeV, and tan β = 56.86.

an arbitrary value of tan β. For simplification of our analysis, we have considered here the case for $\mu < M_N$ so as to avoid the effect of running $h_N(\mu)$ in eqs. (2–5). Our numerical solution shows that for input values of the running masses $m_{\mathrm{top}}(m_{\mathrm{top}})$ = 166.5 GeV, $m_b(m_b)$ = 4.2 GeV (corresponding to $m_{\mathrm{top}}^{\mathrm{Pole}}$ = 175.6 GeV, m_b^{Pole} = 4.7 GeV), and



(b)

Figure 1. Variation of the Yukawa couplings (h_{top}, h_b and h_τ) with absolute energy scale t = ln (μ / 1 GeV) for input values of running quark masses and tan β. (b) $m_{\mathrm{top}}(m_{\mathrm{top}})$, $m_b(m_b)$ = (167.77, 4.55) GeV, and tan β = 52.51

tan β = 56.86, the top-b-τ Yukawa couplings unification scale is obtained at the intermediate scale t = 27.98 (M_I = 1.4 × 10^{12} GeV) as shown in Figure 1a. The variation of tan β does not help much to push M_I at the GUT scale. However with slight variations of the input values, $m_{\mathrm{top}}(m_{\mathrm{top}})$ = 167.77 GeV, $m_b(m_b)$ = 4.55 GeV and tan $\beta \simeq$ 52.51, the Yukawa couplings unification is achieved again at the GUT scale, t = 37.47 (M_I = M_U = 1.87 × 10^{16} GeV) as shown in Figure 1b. Here $m_{\mathrm{top}}(m_{\mathrm{top}})$ = 167.77 GeV is within the uncertainty range, but $m_b(m_b)$ = 4.55 GeV which lies outside the acceptable range, would correspond to m_b^{Pole} = 4.95 GeV if we consider only the one-loop contribution in eq. (14), which still lies in the acceptable range, m_b^{Pole} = (5.0 – 4.7) GeV.

The present finding on top-b-τ Yukawa couplings unification can accommodate the MSSM embedded in GUTs like SO(10) without [2–4,14] and with [9,11] single intermediate symmetry breaking scale (M_I = 10^{11}–10^{13} GeV). With the running top-quark mass, m_{top} = 166.5 GeV, the triviality bound on perturbative consistancy, $h_{top}^2(M_U)/4\pi \leq 1$ has been satisfied for lower values of tan $\beta \geq 1.62$ at the GUT scale $M_U = 1.87 \times 10^{16}$ GeV. Such low value of tan β is particularly important for large hierarchical structure of neutrino masses of three families at low energies in seesaw mechanism [15]. The effect of extra renormalization due to $h_N(\mu)$ on $h_{top}(\mu)$, $h_b(\mu)$, and $h_\tau(\mu)$ for the energy range $t = (t_N - t_U)$ is particularly important for accurate low-energy prediction of small neutrino masses in seesaw mechanism [15].

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References

[1] B Ananthanarayan, G Lazarides and Q Shafi *Phys. Rev.* **D44** 1613 (1991)

[2] V Barger, M S Berger and P Ohmann *Phys. Rev* **D47** 1093 (1993)

[3] N G Deshpande and E Keith *Phys Rev* **D50** 3513 (1993)

[4] P Langacker and N Polonsky *Phys Rev.* **D49** 1454 (1994)

[5] M Olechowski, H Murayana and S Pokorski *Phys. Lett.* **B371** 57 (1996)

[6] E M Freire *Phys. Rev.* **D43** 209 (1991)

[7] M K Parida and A Usmani *Phys Rev.* **D54** 3663 (1996)

[8] N N Singh and S B Singh *Eur. Phys. J.* **C5** 363 (1998)

[9] J Sato *Phys Rev* **D53** 3884 (1996); M Bando, J Sato and T Takahashi *Phys Rev* **D52** 3076 (1995)

[10] D G Lee and R N Mohapatra *Phys. Rev.* **D52** 4125 (1995)

[11] M K Parida *Phys Rev* **D57** 2736 (1998)

[12] N G Deshpande, B Dutta and E Keith *Phys Rev.* **D54** R730 (1996)

[13] F Vissani and A Yu Smirnov *Phys Lett* **B341** 173 (1994); A Yu Smirnov *Nucl Phys* **B466** 25 (1996)

[14] G K Leontaris, S Lola and G G Ross *Nucl Phys.* **B454** 25 (1995)

[15] M K Parida and N N Singh *Phys. Rev.* **D59** 32002 (1998)

[16] G Altarelli hep-hp/9710434

[17] R M Barnett *et al* Particle Data Group *Phys. Rev.* **D54(1)** 303 (1996)

[18] M C Smith and S S Willenbrock *Phys. Rev. Lett.* **79** 3825 (1997)

[19] N Gray, D Broadhurst, W Grafe and K Schilcher *Z. Phys.* **C48** 673 (1990)

[20] R Hempfling and B A Kniehl *Phys. Rev.* **D51** 1386 (1995)

[21] P Langacker and M Luo *Phys. Rev.* **D44** 817 (1991)